(Case Award) and MAE the British Council for a bursary.

References

1. B. SCHWARTZ, *Crit. Rev. Solid State Sci.* 5 (1975) 609.
2. S. M. SPITZER, B. SCHWARTZ and G. D. WEIGLE, *J. Electrochem Soc.* **122** (1975) 397.
3. H. HASEGAWA, K. FORWARD and H. L. HARTNAGEL, *Electron Lett.* **11** (1975) 53.
4. *Idem, Appl. Phys. Lett.* **26** (1975) 567.
5. H. HASEGAWA and H. L. HARTNAGEL, *J. Electrochem Soc.* **123** (1976) 713.
6. J. O. WILLIAMS, P. J. WRIGHT and A. MABBITT, *Mater. Res. Bull* 12 (1977) 1227.
7. S. SZPAK, *J. Electrochem Soc.* **124** (1976) 107.
8. H. GERISCHER, *Electroanal. Chem. Interfacial Electrochem.* **58** (1975) 263.

Received 17 March
and accepted 20 March 1978.

JOHN O. WILLIAMS
PETER J. WRIGHT
M. A. ELMORSI
Edward Davies Chemical Laboratories,
University College of Wales,
Aberystwyth
SALAH E. MORSI
Department of Chemistry
University of Tanta,
Tanta, Egypt

Cracking in layered composites

The progress of a crack through a body or part of a structure will clearly be affected if local variations in the character of the material occur in the path of the crack. A typical example would be the situation when a crack passes through a two-phased material. But almost equally common is the more macroscopic structural variation represented, for example, by a component that has been case-hardened or similarly surface-treated for whatever purpose. Such a situation might be modelled by considering cracking in a sample consisting of a thick plate of some material to which a thinner plate of a different material has been glued. The parameters that will affect the progress of a crack in such a layered structure are the elastic moduli, fracture toughnesses, and relative thicknesses of the two components. A suitable experimental model might be a layered double cantilever beam (DCB) of the kind shown in Fig. 1, and Atkins and Mai [1] have used the Gurney and Hunt analysis [2] to deduce the cracking load for the simplest case of this kind where the thicknesses and elastic moduli of the two components (and therefore their bending stiffnesses, EI) are equal. The fracture toughnesses of the two materials, which determine their cracking loads for a given crack length, are assumed to be different. Fig. 2 shows how the argument, based on a simple energy conservation concept, is developed. The line, OM_1, represents the load/deflection curve for a DCB sample of the tougher of the two components

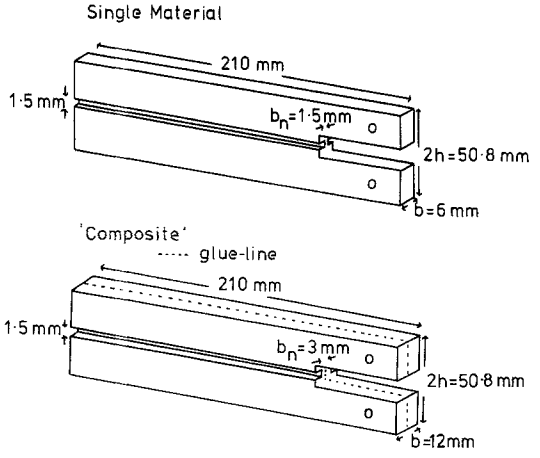

Figure 1 Geometry of double cantilever beam samples.

containing a crack of area A_s. At a critical load P_1 the crack extends in a quasi-static manner and we suppose that when its area reaches A_f the beam is unloaded. In a "reasonably" brittle solid the compliance curve will return to the origin and the fracture work, R_1, can be determined from $OM_1N_1 = R_1(A_f - A_s)$. The cracking of an identical DCB of the less tough material is similarly controlled by the fracture work, R_2, and its Gurney curve is OM_2N_2.

A composite beam containing equal thicknesses of the two materials strongly glued together and tested in the same way will, for the same starting and finishing crack lengths, have compliances identical with those of the two component materials. When the plates are glued together and

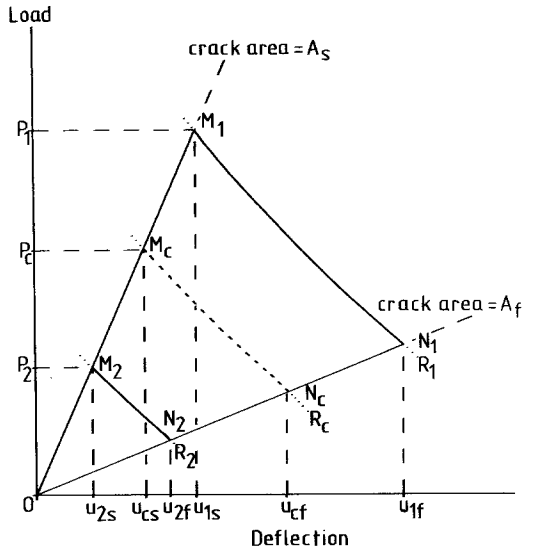

Figure 2 Schematic load/deflection plots for beams of two separate materials and for a composite of the two.

loaded through common loading pins an additional constraint is imposed on the system, which results from the fact that the less tough layer wishes to crack at a deflection u_{2s} whereas the tougher layer would not normally crack until u_{1s}. The glue imparts interfacial tractions to the two layers at the point when the less tough layer would normally crack, and the net result is a compromise behaviour where the propagated crack lengths in the two layers are different from what they would be if free of interfacial tractions. For purposes of illustration, we consider the extreme case for which interfacial slip is completely inhibited, the interfacial glue tractions being capable of forcing the same common compromise crack front in both layers. This means that the load/deflection diagram for each layer becomes identical, (OM_cN_c), crack propagation in both layers from A_s to A_f occurring over the common cross-head movement u_{cs} to u_{cf}. Cracking in the less tough layer has been suppressed by the glue (from u_s to u_{cs}) and cracking in the tougher layer has been promoted at the earlier displacement u_{cs} instead of at u_{1s}. In consequence, during cracking of the composite beam the "tougher" component will be forced to crack at a load below its normal (free) cracking load, and the less tough component will crack at a higher load than normal. In this constrained state, therefore, the first material will appear to have a lower fracture energy than R_1 and the second a higher value than R_2. If there is no net transfer of energy out of the system, and if heating effects can be ignored, the First Law of Thermodynamics dictates that the area OM_cN_c is half the sum of the areas OM_1N_1 and OM_2N_2. And since these areas are directly related to R we can write

$$OM_cN_c = R_c(A_f - A_s) = \frac{(R_1 + R_2)(A_f - A_s)}{2}$$

or

$$R_c = \tfrac{1}{2}(R_1 + R_2)$$

R may be written in terms of the strain energy release rate,

$$R = G = \frac{P^2}{2}\left(\frac{dC}{dA}\right)_A$$

where $(dC/dA)_A$ is the rate of change of compliance with crack area at a given crack area A. For this composite system, since dC/dA is the same for both components and for the composite, we see that

$$P_c^2 = \tfrac{1}{2}(P_1^2 + P_2^2)$$

We conclude that the cracking load for the composite beam, P_c, is given by

$$P_c = [\tfrac{1}{2}(P_1^2 + P_2^2)]^{1/2}$$

and we note that this value of P_c is *always greater* than the simple mixture-rule sum, $\frac{1}{2}(P_1 + P_2)$. This result can be equally well expressed in terms of the relevant critical stress intensity factors, i.e. for two otherwise equal specimens,

$$K_{\text{composite}} = [\tfrac{1}{2}(K_1^2 + K_2^2)]^{1/2}$$

This clearly throws doubt on the recent suggestion of Parvin and Williams [3] that failure under mixed plane strain/plane stress conditions can be accounted for by adding together, in proportion to the relevant fracture surface areas, the known values of fracture toughness for plane strain and plane stress cracking – i.e. a mixture-rule sum. According to the Atkins and Mai model the cracking load of a composite is the same as the volume fraction sum only when there is no bonding between the components.

The kind of effect that we have just described is often referred to, in respect of composite materials, as a synergistic effect. The experiments we now discuss were intended to demonstrate that a synergistic effect can exist in simple composites. Before reporting the results, however, a word of caution is necessary. The synergism arises from addition of work areas when glueing forces a common crack front. Consequently a glue has to be found which will perform this task satisfactorily. In addition, care must be taken that like is being compared with like, particularly if the fracture toughness of a solid varies with crack velocity. It has been shown [2] that the ratio of crack velocity to cross-head velocity depends inversely on R. Thus, other things being equal, the common crack velocity of the glued laminate (cracking with an apparent R_c in each layer) will be *greater* than the crack velocity displayed by the less tough (R_2) layer when free, and *less* than that of the free R_1 layer. If, in practice, R varies with crack velocity, and hence cracking loads vary with crack velocity, it would not be correct to compare the load/deflection diagrams of free and glued layers obtained at the same testing machine cross-head speed; rather, comparisons should be made under conditions of similar crack velocities, produced if necessary by different cross-head speeds. In what follows, such complications have been avoided or accommodated.

In order to test the Atkins and Mai model we have carried out crack propagation experiments on DCB samples made by glueing together pairs of transparent brittle polymers. A suitable pair of polymers would have different fracture toughness but a similar mode of crack propagation. PMMA is an obvious choice for one of the pair, partly because it has been exceptionally well characterized, and partly because in a DCB test the crack propagates slowly and smoothly with a falling-load characteristic. Preliminary experiments with polyester resin as the other member of the pair were unsuccessful because the pure resin, although much more brittle than PMMA, cracks in a stick–slip mode with very high crack velocity. Cracking in the composite beam was as slow as in PMMA, however, and no valid comparison of the measured K_c values could be made because of this difference in crack velocity. Other trials with polycarbonate were also unsatisfactory because, although it is much tougher than PMMA, cracking in polycarbonate was accompanied by gross plastic flow, and again our comparisons were invalid. The experiments we describe here were on PMMA/PVC beams. PVC is by no means ideal, for although a manufacturer's sample cracked in an unstable brittle fashion, material cut from a larger plate purchased from commercial stock usually failed instead by a pop-in mechanism. Since the pop-in phenomenon occurred at the same loads as brittle crack growth, however, we believe that our analysis is valid.

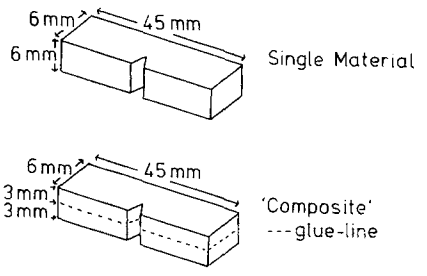

Figure 3 Geometry of Charpy impact test samples.

Pairs of beams were glued together by means of a low-viscosity cyanoacrylate adhesive. The resulting glue-line was very thin, the polymerized adhesive was rigid, and the sample remained transparent so that the crack front could be easily observed.

The first experiment was to test the effect of the adhesive bond on the total energy absorption during fracture. Notched Charpy impact tests were carried out, on samples of the kind shown in Fig. 3, for the separate polymers and for the composite beam. In the composite the crack was made to travel, as shown, normal to the glue line simultaneously in both components. Table I shows that the measured work of fracture for the composite is slightly lower than a mixture-rule average of the separate material values. There is therefore no substantial contribution to fracture work from deformation or failure events in the adhesive and the interfacial regions (i.e. in the "glue-line").

TABLE I Work of fracture from Charpy tests*

Separate materials (kJ m^{-2})		Composite (kJ m^{-2})	
PMMA	PVC	Mixture rule average	Measured values
1.2 ± 0 2	1.6 ± 0.3	1.4	1.3 ± 0.2

*Results are averages of about five tests.

TABLE II K_c values for PMMA

PMMA (separate samples)			PMMA/PMMA "Composites"		
K_c(MN m$^{-3/2}$)	Number of samples	Number of results	K_c(MN m$^{-3/2}$)	Number of samples	Number of results
1.09 ± 0.04	6	59	1.02 ± 0.04	2	19

Critical stress intensity factors were measured on the DCB samples shown in Fig. 1 after starter cracks were induced at the roots of machined notches by means of a Stanley knife blade. Values of the critical stress intensity factor were obtained at various crack lengths by means of the Gross–Srawley expression [4]

$$K_c = \frac{3.4(c/h + 0.7)}{\sqrt{\{bb_n h(1 - \nu^2)\}}}$$

where geometric symbols are defined in Fig. 1, c is the crack length measured from the loading axis, and ν is Poisson's ratio. Since the composite beams were twice the thickness of the separate material samples and contained an adhesive joint we first assessed the effects of these differences by testing control composite beams made by glueing two identical PMMA samples together. Table II shows the results of these tests. The thicker PMMA "composite" with the glue-line has a slightly smaller value of K_c, and again therefore it appears that the extra thickness and the presence of the glue-line should not, of themselves, affect the crack behaviour of the PMMA/PVC composites.

Our measured values of K_c appear to be reasonably independent of crack length for PMMA, PVC and the composite, as shown in Fig. 4, and mean values are given in Table III. It can be seen that our mean composite K_c is much higher than the mixture-rule average and is very close to the value predicted by the model of Atkins and Mai. Comparing the measured value with the mixture rule average, it may be shown that the difference between the means is very significant indeed, being of the order of nine times its standard error.

A simple mechanics treatment shows that when a crack propagates in a layered structure the cracking load, P_c, is given, in terms of the cracking loads, P_1 and P_2, for the separate materials, by

$$P_c = [\tfrac{1}{2}(P_1^2 + P_2^2)]^{1/2}$$

This value is always greater than a linear sum. A similar relationship may be written for the critical stress intensity factor of the composite, and this has been verified experimentally for composite beams of PMMA and PVC. A more detailed theory has also been developed for composite beams in which the component thicknesses and moduli (and therefore their bending stiffnesses, EI) are different as well as their toughnesses, and this will be published in due course. We also wish to study the effects of changing the rigidity of the adhesive

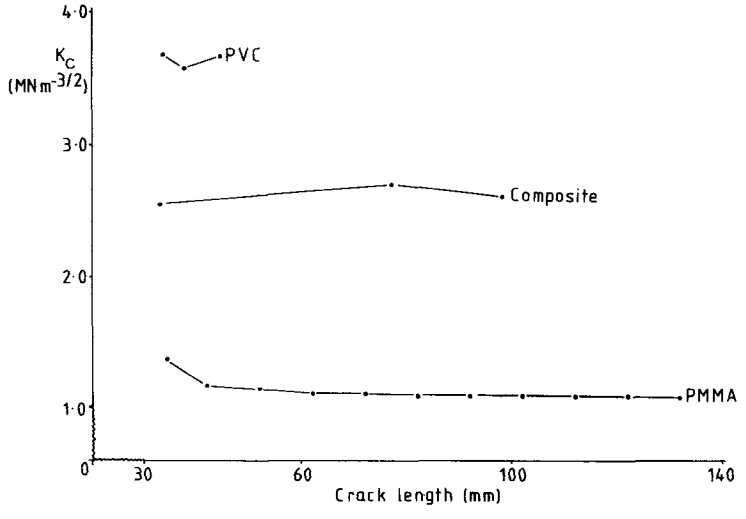

Figure 4 Critical stress intensity factor, K_c, versus crack length for PMMA, PVC and composite beams.

TABLE III K_c values for PMMA, PVC and composite beams

Separate materials				Composite				
PMMA*	PVC			Measured value			Mixture rule average	Value predicted by Atkins & Mai theory
K_c(MN m$^{-3/2}$)	K_c(MN m$^{-3/2}$)	Number of samples	Number of results	K_c(MN m$^{-3/2}$)	Number of samples	Number of results	K_c (MN m$^{-3/2}$)	K_c (MN m$^{-3/2}$)
1.09 ± 0.04	3.56 ± 0.20	9	13	2.68 ± 0.08	5	9	2.32	2.63

*See Table II

joint. We are interested in applying this model to other materials or structures, particularly to the case of hybrid, or mixed-fibre composites in which fracture loads and fracture strains are frequently found to be higher than expected.

References

1. A. G. ATKINS and Y. W. MAI, *Int. J. Fracture* **12** (1976) 923.
2. C. GURNEY and J. HUNT, *Proc. Roy. Soc.* **A299** (1967) 508.
3. M. PARVIN and J. G. WILLIAMS, *Int. J. Fracture* **11** (1975) 963.
4. J. E. SRAWLEY and B. GROSS, NASA TN D-3820 (1967).

Received 17 May
and accepted 26 May 1978.

F. J. GUILD
B. HARRIS
School of Materials Science,
University of Bath, Bath, UK
A. G. ATKINS
Delta Materials Research Ltd.
Ipswich, UK